MPA skills in Algebra

Key Processes in Algebra

Introduction

Algebra in Key Stages 3 and 4 is based on the generalisations of relationships familiar from basic number. It is developed to include the use of equations, formulae and identities, and sequences, functions and graphs. Algebra is purposeful when pupils encounter sufficiently complex situations where objects or relationships require representation in symbolic or graphical form. These occur frequently when describing generalisations underlying particular relationships. To use and make sense of algebra, pupils need opportunities to relate it to their knowledge of the arithmetical operations. Suitable contexts for algebraic representation may come from within mathematics (for example, exploring number patterns and puzzles, or finding areas of shapes), by linking with other subjects or from real-life applications. It should include use of ICT, such as graph-plotting and spreadsheet software to explore functions.

Representing

Representing a situation places it into a mathematical form that enables it to be worked on. In algebra this might mean trying out and choosing between different diagrammatic, graphical and symbolic forms arising from looking at the problem or situation from different points of view. Aspects of representing within algebra include:• identifying assumptions, variables and relationships in order to create a mathematical model• developing understanding of algebraic conventions; for example, conventions of writing terms and expressions, coordinate points and equations of lines, vectors and magnitude of vectors• constructing algebraic expressions, equations, formulae and identities; for example, understanding and using signs such as =, _, < and > to represent relationships between variables• choosing appropriate algebraic representation of such relationships, using knowledge of equivalence forms (e.g. tables, functions and graphs), so that the context can be analysed and the solution communicated• choosing the tools most appropriate to the represent the mathematics drawn from the situation; for example, a graphical calculator or a spreadsheet.As well as giving point to the subject, experience of algebraic representation is crucial if pupils are to understand and use precise algebraic language. Giving explicit attention to this helps them to understand the conventions for using letter symbols and constructing algebraic expressions. It can also give pupils insights into algebraic structure and order of operations, needed when transforming or interpreting symbolic and graphical representations.

Analysing – using mathematical reasoning

Algebra as a tool lies at the heart of much mathematical reasoning. Pupils need opportunities to experience the power of algebra in expressing generality. This includes:• identifying and describing numerical patterns and relationships, both symbolically and graphically• making connections with arithmetical operations and with equivalent algebraic forms when transforming expressions and equations• making connections between sequences, functions and graphs, and exploring the effects of varying values• making generalisations, explaining and proving, and relating results to the context of the problem.

Analysing – using appropriate mathematical procedures

Using appropriate procedures involves manipulating expressions, equations and graphs, using and applying techniques and accurate notation, and monitoring the accuracy of methods and solutions. Appropriate procedures in algebra include:• generating equivalent expressions and equations, including a simplified form• factorising and expanding expressions and equations• solving equations exactly and approximately• manipulating formulae, including changing the subject of the formula• substituting values into equations and formulae; for example, evaluating a formula to convert temperature in degrees C to degrees F.Algebra in Key Stages 3 and 4 is generalised arithmetic. It requires understanding of the commutative, associative and distributive laws as they apply to the number operations, and of relationships between operations, including inverses. Pupils can be supported to generalise the rules with letters in place of numbers; for example, ab = c implies: ba = c, b=c/a, a=c/b, 2ab=2c, 2ab+1 = 2c+1Taking an exploratory approach to transforming algebraic expressions and equations, where pupils are regularly asked to write expressions in different ways (‘find as many ways as you can’), builds their algebraic skills. Pupils:• gain confidence in manipulating expressions into different equivalent forms• gain insights into which of a range of possible transformations will be both valid and efficient as a next step, for example in solving an equation or rearranging a formula• develop increasing fluency with algebraic manipulation without being rule-bound and, when the steps in a procedure are not obvious, are able to resolve difficulties for themselves.

Interpreting and evaluating

Aspects of interpreting and evaluating in algebra include:• relating numerical results, such as the solution of an

equation, to the context under consideration• interpreting general statements or conclusions expressed in algebraic form (e.g. an expression or formula) and considering their significance• recognising the difference between numerical evidence and algebraic proof• interpreting graphs and graphical features such as points of intersection, gradients and the general shape of a graph• evaluating different approaches; for example, where another pupil has represented the problem or approached its solution in a different way.

Communicating and reflecting

Aspects of communicating and reflecting in algebra include:• recognising and using the fact that algebraic language (symbolic and graphical) is a powerful form of communication for expressing the steps in an argument or the conclusions of an enquiry• considering alternative approaches; for example, comparing algebraic, graphical and numerical approaches to tackling a problem• making links to related problems or to different problems with a similar structure